UK Junior Mathematical Olympiad 2004

Organised by The United Kingdom Mathematics Trust

Tuesday 15th June 2004

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

A1 Write $1 + \dfrac{1}{1 + \dfrac{1}{3}}$ as a decimal.

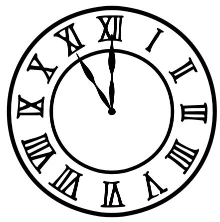
A2 Gus is older than Flora. Alessandro is older than Zara but younger than Flora. Oliver is younger than Gus but older than Zara. Yvette is younger than Gus. Alessandro is older than Oliver. Flora is younger than Yvette. Which of these six friends is the youngest?

A3 On Monday the Pied Piper caught 1000 rats in a city. On Tuesday he caught 10% fewer than on Monday. On Wednesday he caught 20% more than on Tuesday. On Thursday he caught 30% fewer than on Wednesday. On Friday he rested. How many rats did he catch in total that week?

A4 Three brothers and a sister shared a sum of money equally among themselves. If the brothers alone had shared the money, then they would have increased the amount they each received by £20. What was the original sum of money?

A5 Walking up a steep hill, I pass 10 equally spaced street lamps. I take 5 seconds to walk from the first lamp to the second lamp, 6 seconds from the second lamp to the third, and so on, with each time increasing by 1 second as I slow down. How long do I take to walk from the first lamp to the last?

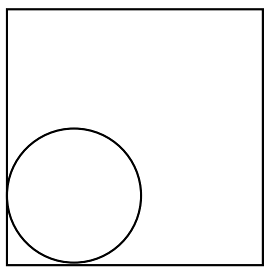
A6 Professor Brainstorm's clock gains 16 minutes every day. After he has set it to the correct time, how many days pass before it next tells the correct time?



A7 The square of a number is 12 more than the number itself. The cube of the number is 9 times the number. What is the number?

A8 A large pan contains a mixture of oil and water. After 2 litres of water are added to the original contents of the pan, the ratio of oil to water is 1:2. However, when 2 litres of oil are added to the new mixture, the ratio becomes 2:3. Find the original ratio of oil to water in the pan.

A9 A circle of radius 1 rolls without slipping round the inside of a square of side 4. Find an expression involving π for the exact number of revolutions the circle makes by the time it first returns to its original position.



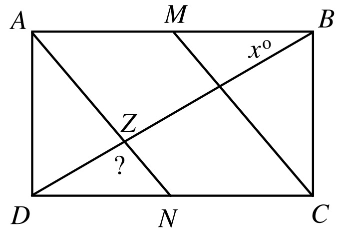
A10 The Famous Five have been given 20 sweets as a reward for solving a tricky crime. They have agreed that the oldest of them must receive more than the next oldest, who must receive more than the next oldest, and so on. Assuming that each of the five gets at least one sweet, in how many different ways can they share the sweets?

Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief ‘answers’).

B1 In the rectangle *ABCD*, *M* and *N* are the midpoints of *AB* and *CD* respectively; *AB* has length 2 and *AD* has length 1.

Given that $\angle ABD = x°$, calculate $\angle DZN$ in terms of x.



B2 Three identical rectangular cards can be placed end to end (with their short sides touching) to make rectangle A, and can be placed side by side (with their long sides touching) to make rectangle B. The perimeter of rectangle A is twice the perimeter of rectangle B.

Find the ratio of the length of a short side to the length of a long side of each card.

B3 The solution to each clue of this cross-number is a two-digit number. None of these numbers begins with zero.

Complete the crossnumber, stating the order in which you solved the clues and explaining why there is only one possibility at each stage.

Clues Across

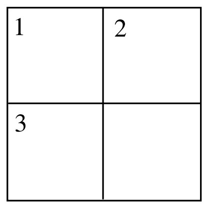
1. Multiple of 3

3. Three times a prime

Clues Down

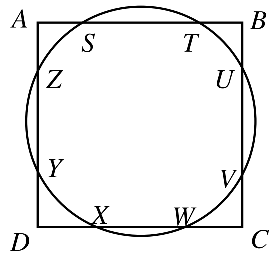
1. Multiple of 25

2. Square



B4 In the square *ABCD*, *S* is the point one quarter of the way from *A* to *B* and *T* is the point one quarter of the way from *B* to *A*. The points *U*, *V*, *W*, *X*, *Y*, *Z* are defined similarly. The eight points *S*, *T*, *U*, *V*, *W*, *X*, *Y*, *Z* lie on a circle, whose centre is at the centre of the square.

Determine which has the larger area: the square *ABCD*, or the circle.



B5 On an adventure holiday five children, called *A*, *B*, *C*, *D*, *E*, all take part in five competitions, called *V*, *W*, *X*, *Y*, *Z*. In each competition marks of 5, 4, 3, 2, 1 are awarded for coming 1st, 2nd, 3rd, 4th or 5th respectively. There are no ties for places.

Child *A* scores a total of 24 marks, child *C* scores the same in each of four competitions, child *D* scores 4 in competition *V*, and child *E* scores 5 in *W* and 3 in *X*. Surprisingly, their overall positions are in alphabetical order.

Show that this information is enough to find all the scores, and that there is only one solution. Give the marks scored by each child in each competition by filling in a copy of this table.

	V	*W*	*X*	*Y*	*Z*	Total
A						
B						
C						
D						
E						

B6 Suppose that the diagram is to be completed so that each white square contains a different whole number from 1 to 12 inclusive and also so that the four numbers in the set of squares along each edge have the same total.

In how many different ways can the diagram be completed correctly?